9.2 Usage Characterization for Simple Repairs

As explained in Section 9.1, there are two reasons for characterizing the usage for a damage tolerance analysis: (a) to establish the residual strength requirement and (b) to grow the crack in the sub-critical crack growth analysis. The maximum loading conditions and their frequency determine the loading (residual strength) requirement that the structural element must withstand without failure. Typically the more frequently occurring loading conditions are those responsible for growing a crack from its initial size to failure.

For a residual strength analysis of a structural repair, one would want to characterize the maximum loading condition that the structure might experience in its anticipated service life. A simple choice might be based on returning the load carrying capability of the cracked structure to the original ultimate load carrying capability of the structural member without a crack. Section 9.7 describes in more detail the methods for conducting a residual strength analysis.

For a sub-critical crack growth analysis, one is more typically interested in characterizing the average per flight loading conditions that will be experienced by the cracked or repaired structure. It is the relatively large, frequently occurring load excursions that drive the crack growth process. From a repair analysis standpoint, it is important that the analyst know what are the sources of large (and frequently occurring) stress excursions and have some indication of the maximum to minimum stress ratios as well as frequency of these excursions on a per flight (or per flight hour) basis.

The more critical the repair, the more important is the definition of the specifics of per flight average loading conditions for a life analysis. If one can identify those loading conditions that affect the rate at which cracks grow in a given structural member, then simple calculations can be made to obtain first order estimates of this member's structural life. While first order estimates can be questioned from an absolute sense, such estimates, when used in a relative sense, can provide the necessary guidance for designing a repair, or releasing an individual aircraft for flight.

In the following subsections, a sub-critical crack growth analysis approach, which was introduced in Section 5.2.5, is further described and justified for its application for repair analysis. In Section 9.3, an example analysis of three transport wing stress histories is utilized to illustrate how a generic stress history for a given structural member could be employed to estimate the life at any given location in that member.

9.2.1 Variable Amplitude Crack Growth Behavior

Many airframe loading conditions are sufficiently repetitive over a number of flight (~100 to 500 flights) that the crack growth damage accumulates in a relatively continuous manner. Figure 9.2.1 describes two examples of experimental crack growth data generated under typical flight-by-flight loadings involving multiple missions. Figure 9.2.1a represents the behavior experienced at a hole subjected to a fighter wing stress history and Figure 9.2.1b represents the behavior observed at a hole subjected to a bomber aircraft wing stress history. Both behaviors illustrate the regular and relatively continuous crack growth pattern exhibited by many flight-by-flight histories.

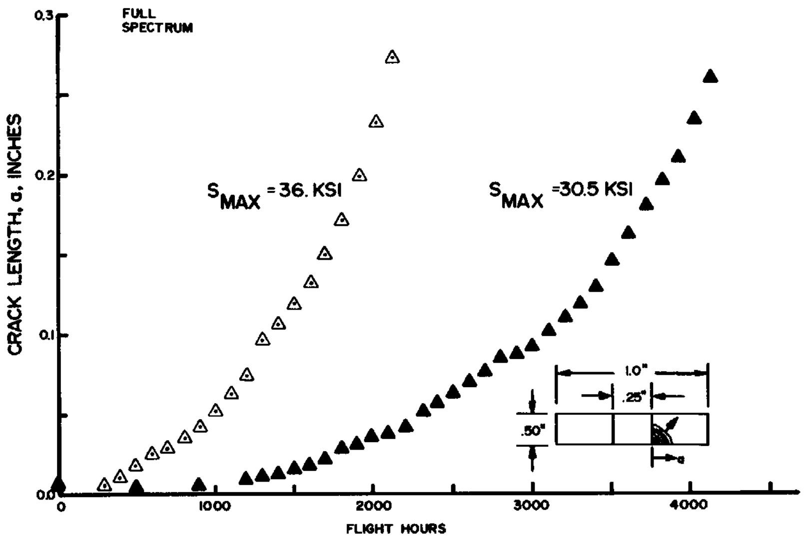

Figure 9.2.1a. Experimental Propagation Behavior of Corner Crack with Full F-4E/S Wing Spectrum (68000 cycles/1000 flight hours) Scaled to Two Stress Levels (36 and 30.5 ksi).

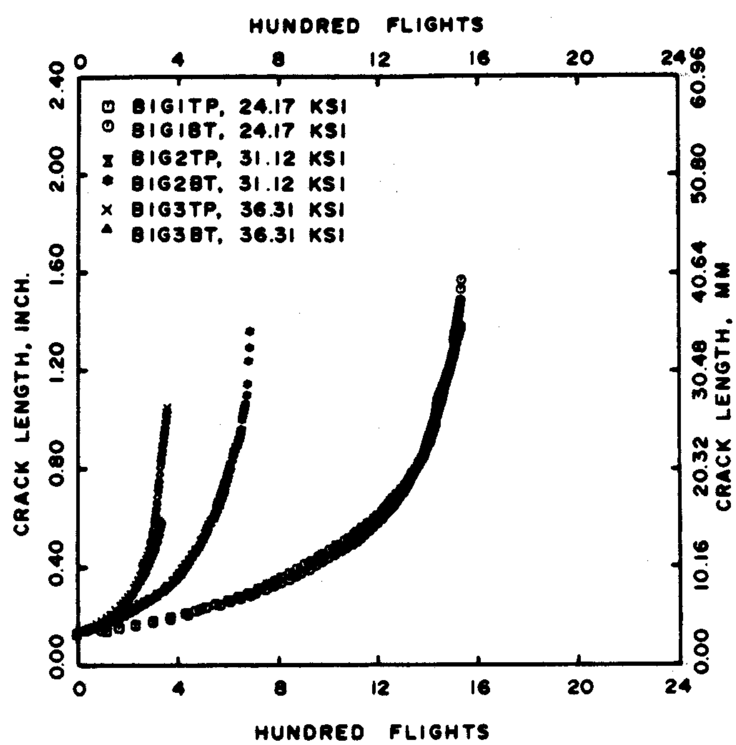

Figure 9.2.1b. Experimental Flight-By-Flight Fatigue Crack Growth Behavior for a B-1A Wing Spectrum Scaled to Three Levels (24.17, 31.12, and 36.31 ksi).

As a result of the regularity of such flight-by-flight induced crack growth behavior, there was a recognition as early as 1963 that aircraft stress histories can induce crack growth behavior similar to constant amplitude behavior. This early recognition has led to a number of schemes for utilizing limited information to characterize the behavior of cracks in aircraft structure. These schemes all focus on the translation of variable amplitude crack growth life data to variable amplitude crack growth "rate" data so that the simple analysis schemes for constant amplitude loadings can be used to establish life estimates. These replace the more complicated numerical, computer-based algorithms used for a load interaction, cycle-by-cycle analysis of the complete stress history.

The translation of the variable amplitude crack growth life data to that of variable amplitude crack growth rate data follows most of the procedures used to convert constant amplitude crack growth life data to constant amplitude crack growth rate data (see Subsection 7.2.2 and Figure 7.2.9, which is repeated as Figure 9.2.2). The major differences between describing variable amplitude rate behavior and constant amplitude rate behavior is in the choice of the rate variable and the characterizing stress history parameter. In variable amplitude descriptions, the crack growth rate may be described as rate per flight, rate per flight hour, or rate per cycle. Also, since the magnitude of the individual stress events in the stress history is a random variable the characteristic stress parameter that described the history is a statistical measure of the individual events in the history.

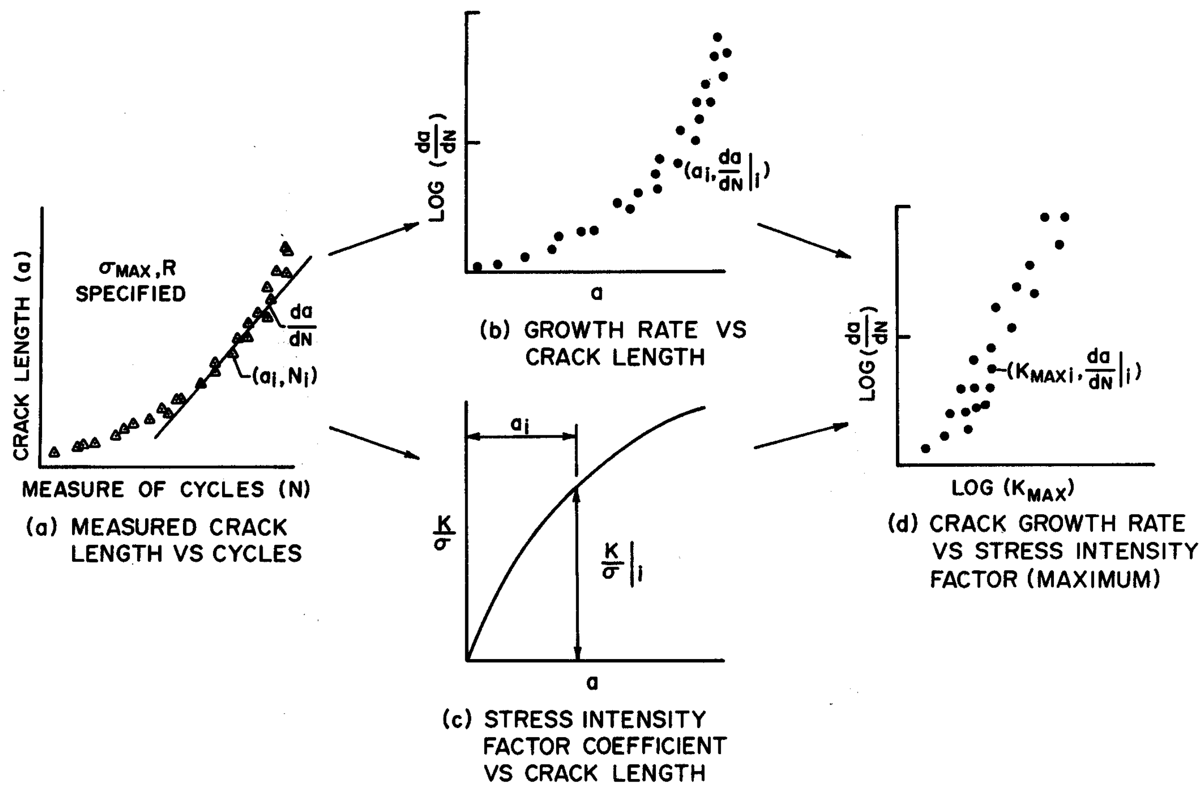

Figure 9.2.2. Method for Reducing Fatigue Crack Growth Life Data to Fatigue Crack Growth Rate Data

Figure 9.2.3 describes a variable amplitude crack growth rate behavior ($da/d(Flight)$) as a function of a spectra dependent characteristic stress-intensity factor ($\overline{K}$) for two transport wing histories. The K is calculated based on the formula

$$\overline{K} = \overline{\sigma} \cdot \left(\frac{K}{\sigma} \right) \tag{9.2.1}$$

where K/σ is the stress-intensity factor coefficient for the geometry and $\overline{\sigma}$ is the characteristic stress parameter, here chosen as the root mean square (RMS) of the maximum stresses in the history, i.e.

$$\overline{\sigma} = \overline{\sigma}_{\max} = \left[\sum_{i=1}^{N} \frac{\left(\sigma_{\max_i}\right)^2}{N} \right]^{\frac{1}{2}} \tag{9.2.2}$$

In equation 9.2.2, N is the number of stress events, and $\sigma_{max(i)}$ denotes the maximum stress for the i^{th} stress event.

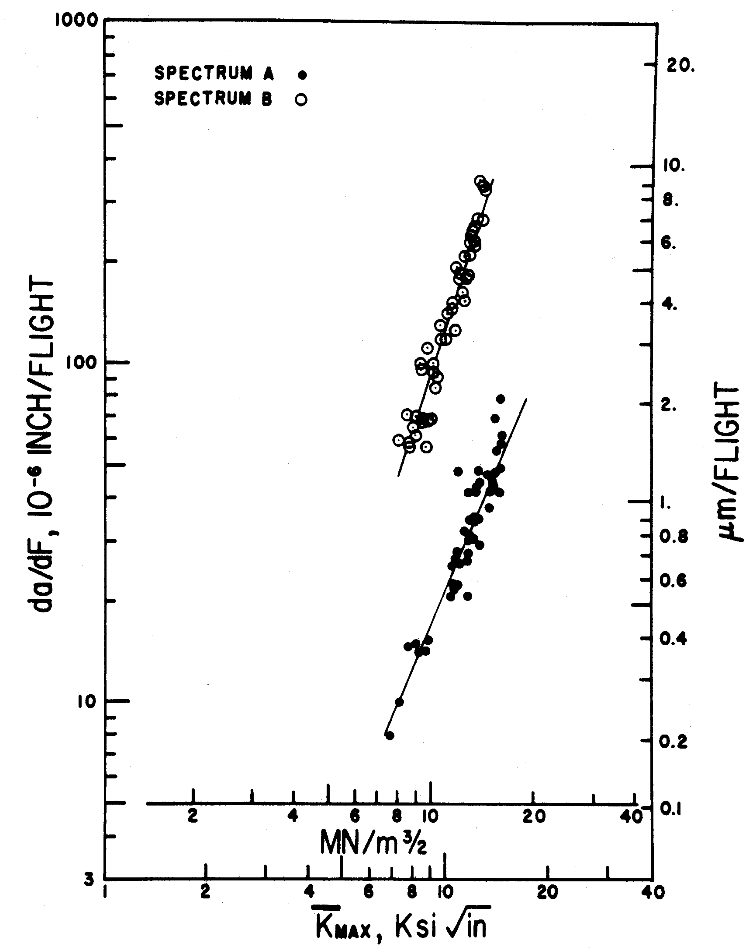

Figure 9.2.3. Fatigue Crack Growth Rate Data for Two Transport Spectra (A = Upper Wing, B = Lower Wing)

It is seen from Figure 9.2.3 that the crack growth rate (on a per-flight basis) behavior for the two spectra might be described by a power law equation of the form

$$\frac{da}{dF} = C\overline{K}^{\,p} \tag{9.2.3}$$

The corresponding image integration equation can be expressed as

$$F = \int_{a_0}^{a_f} \frac{da}{C\overline{K}^{\,p}} \tag{9.2.4}$$

or as

$$a = a_0 + \sum_{j=1}^{F} \Delta a_j \tag{9.2.5}$$

where F is the number of flights required to grow the crack from a_0 to a, and where Δa_j is evaluated for the current crack length using Equation 9.2.3. The coefficients C and p are evaluated using least squares procedures applied to data of the type shown in Figure 9.2.3.

The value of the data shown in Figure 9.2.3 and its description with a simple equation, e.g. Equation 9.2.3, is that parametric studies can be conducted in a relatively simple manner. Such parametric studies could cover other ranges of crack length for the same geometry, other structural geometries, and other stress magnification factors applied to the same spectra.

9.2.2 Other Methods for Generating Rate Descriptions

Translating experimental crack growth life data to flight-by-flight crack growth rate only provides one method for generating the flight-by-flight power law growth rate relationship given by Equation 9.2.3. The power law rate equation can also be generated using two different analytic methods. One popular analytical method is to calculate the RMS range and maximum parameters and to substitute these parameters into a constant amplitude - stress ratio equation. This method results in a single curve that describes the effects of this stress combination. The following example illustrates the procedure.

EXAMPLE 9.2.1 <u>RMS Power Law Analysis</u>

The constant amplitude crack growth equation for a particular alloy is given by

$$\frac{da}{dN} = 8.63x10^{-8} K_{eff}^{2.347}$$

where the effective stress-intensity factor has been determined to be of the form:

$$K_{eff} = \left[K_{\max} - \frac{3}{(1-R)^{0.35}} \right] \cdot (1-R)^{0.696}$$

When the values of maximum stress (σ_{max}) and stress range $(\Delta\sigma)$ for a given constant amplitude loading are known, these values are used with the stress-intensity factor coefficient (K/σ) for the geometry of interest to generate K_{max},

$$K_{\max} = \sigma_{\max} \cdot \left(\frac{K}{\sigma}\right)$$

and with the stress ratio formula

$$R = 1 - \frac{\Delta\sigma}{\sigma_{\max}}$$

to generate the parameters defined in the growth equation. To obtain the power law relation that would result for a flight-by-flight spectrum, the RMS values are substituted into the equations.

If the RMS range ($\Delta\sigma$) is 4.45 ksi and maximum (σ_{max}) stresses for a given stress history are 12.30 ksi, the RMS stress ratio (R) is given by

$$\overline{R} = 1 - \frac{\Delta\overline{\sigma}}{\overline{\sigma}_{max}} = 1 - \frac{4.45}{12.30} = 0.640$$

and the RMS maximum stress-intensity factor is

$$\overline{K}_{\max} = \overline{\sigma}_{\max}\left(\frac{K}{\sigma}\right) = 12.3\left(\frac{K}{\sigma}\right)$$

The growth rate is

$$\frac{da}{dN} = 8.63x10^{-8}\,\overline{K}_{eff}^{2.347}$$

where

$$\overline{K}_{eff} = \left[\overline{K}_{max} - \frac{3}{(1-0.640)^{0.35}}\right] \cdot (1-0.640)^{0.696} = \left[\overline{K}_{max} - 4.29\right] \cdot 0.491$$

Notice that the rate *da/dN* is given on a per cycle basis so one must multiply this rate by the average number of cycles per flight or flight hour to obtain the corresponding average growth rate per flight or flight hour.

A method that substitutes RMS (or other statistically derived) parameters into constant amplitude equations has one major limitation. This limitation is that load interaction effects (retardation or acceleration) are ignored. Thus, the analyst must be wary of comparisons between spectra when using this method, since it will only provide first order approximations of spectra effects.

It is possible to account for load interaction effects with a cycle-by-cycle analysis, but as indicated above, the processing of the complete stress history requires extensive numerical analysis. An approach was suggested in the early 1970's for processing a limited portion of a stress history with a cycle-by-cycle analysis for the purpose of generating crack increments at several crack lengths. Most of the details for generating crack increments for such an analysis were discussed in subsection 5.2.5 relative to Figures 5.2.10 and 5.2.11. Figure 5.2.10 is repeated here as Figure 9.2.4, and the corresponding crack growth rate data is presented in Figure 9.2.5. The choice of methods that one might employ for the cycle-by-cycle analysis is dictated by the success that a given analysis has had in predicting crack growth behavior of the type under consideration. In Section 9.3, a detailed example of an analytical analysis of the crack growth behavior (life and rate) of three transport wing stress histories is conducted. This example should provide additional insight into how the simplified rate method can be used to assess spectra and their differences.

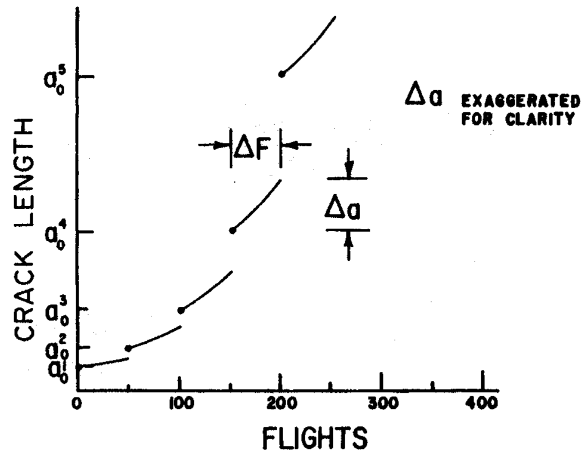

Figure 9.2.4. Crack Incrementation Scheme Based on Cycle-by-Cycle Crack Growth Analysis

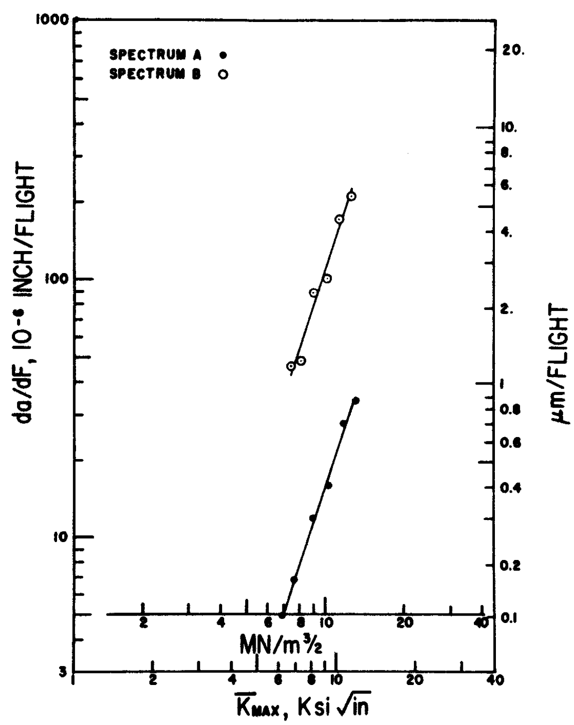

Figure 9.2.5. Crack Growth Rate Description of Crack Incrementation Data for Two Transport Wing Stress Histories (*ΔF* = 50 Flights)

9.2.3 Power Law Descriptions

A number of experimental and analytical investigations have revealed that the flight-by-flight crack growth rate behavior of military aircraft can be described with a power law relationship (Equation 9.2.3). Specifically, the stress histories considered were developed to facilitate the design of a new structure or an analysis of an in-service aircraft for force management purposes. As such, these stress histories represented an expected average usage based on a force wide composite mission mix; most of the stresses in these histories repeated after an application of a large block of flights or flight hours. None of the histories involved any major mission change during the expected life of the aircraft. For these histories, one might say that the operations today will be like the operation next year or five years from now.

Nevertheless, the generalized observations of power law flight-by-flight crack growth rate behavior here are immediately applicable to the study of parameters affecting structural repair. Thus, the results of these studies are summarized in Tables 9.2.1 and 9.2.2 for bomber/transport behavior and for fighter/attack/trainer behavior, respectively. Table 9.2.1 presents the coefficients for a crack growth rate per flight type equation, while Table 9.2.2 presents the coefficients for a crack growth rate per flight hour type equation.

The reader can note from Table 9.2.1 that the exponent p for bomber/transport aircraft wing stress histories only varies from about 3.0 to 3.5; Table 9.2.2 indicates a wider variation in the exponent for the aircraft and conditions indicated ($2.2 \leq p \leq 3.7$). Based on a close analysis of the results, it can be said that the largest variations in the exponent p are generated due to the wide variations in spectrum content (load magnitude and frequency).

Table 9.2.1. Bomber/Transport Behavior

Aircraft	History	Flights/Block	$\bar{\sigma}_{max}$ (ksi)	C^{+}	p	Aluminum Alloy
B1-B	Wing pivot	100	27.3	4.91×10^{-8}	3.025	2219-T851
C-5A	Upper wing	100	11.7	1.70×10^{-8}	3.111	7075-T651
C-5A	Lower wing	300	12.3	1.05×10^{-7}	3.183	7075-T651
B-52D	Lower wing	200	16.4	2.61×10^{-8}	3.529	7075-T651
KC-135	Proof test, Lower wing	200	17.8	5.97×10^{-9}	3.454	7178-T6
KC-135	Lower wing	200	18.4	1.01×10^{-8}	3.338	7178-T6

+ inch/flight, ksi $\sqrt{in}$

Table 9.2.2. Fighter/Attack/Trainer Behavior (Based on 1000 Flight Hour Block Spectra)

Aircraft	History	C^{+}	p	Aluminum Alloy
T-38	Lower wing (baseline)	$2.66\text{x}10^{-8}$	2.678	7075-T651
T-38	Lower wing (severe)	$1.07\text{x}10^{-8}$	3.152	7075-T651
T-38	Lower wing (mild)	$5.32\text{x}10^{-9}$	2.460	7075-T651
F-4	Lower wing (baseline)	$1.68\text{x}10^{-8}$	2.242	7075-T651
F-4	Lower wing (high stress baseline)	$1.77\text{x}10^{-8}$	2.242	7075-T651
F-4	Lower wing (severe)	$1.76\text{x}10^{-8}$	2.395	7075-T651
F-4	Lower wing (mild)	$5.77\text{x}10^{-9}$	2.395	7075-T651
F-16	Lower wing (mix)	$6.92\text{x}10^{-10}$	3.62	7475-T7351
F-16	Tail (mix)	$1.33\text{x}10^{-10}$	3.67	7475-T7351
F-16	Lower wing (air to air)	$1.07\text{x}10^{-9}$	2.905	7475-T7351
F-16	Lower wing (air to ground)	$8.94\text{x}10^{-11}$	3.464	7475-T7351

+ inch/flight, ksi $\sqrt{in}$

Before employing a flight-by-flight crack growth rate type analysis to estimate the life of a repair, the analyst should be concerned with the adequacy of such an analysis. The most important part of the analysis is the definition of the stress history that the repaired member will experience in the future. If the history is anticipated to be statistically repetitive as a function of time-in-service then the results from a flight-by-flight rate analysis will be comparable to a cycle-by-cycle analysis.

If the mission type or mix is expected to change significantly as a function of time, then projecting a predefined rate of crack growth without detailed consideration of how the damage will be changing could lead to non-conservative errors. One method for addressing mission type or mix changes is to utilize one rate curve before the time of change and another rate curve subsequently. A more exact method for addressing mission changes is by using a cycle-by-cycle crack growth analysis applied to the stress history that accounts for the changes.

Rate methods have one inherent problem: they tend to minimize the effects of the infrequently applied large loads. These large loads will cause retardation effects and tend to slow the growth process (if, in application, failure is not induced). Thus rate methods will normally predict somewhat shorter (more conservative) lives than the cycle-by-cycle analyses.

Based on Tables 9.2.1 and 9.2.2 the analyst should note that the crack growth rate equation is a function of material, location, and usage. An equation generated for the horizontal tail should not be used for the vertical tail (nor wing); an equation generated for air-to-ground operations should not be utilized for air-to-air operations.